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Frequency domain and reliability analyses of concrete chimney under vortex-induced vibration

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SUMMARY:
Vortex-induced force can be modelled as a random process described by its power spectral density and cross-power density functions. In this paper, frequency domain analysis of a concrete chimney subjected to vortex-induced excitations, along with reliability analysis is presented. Both modal and direct spectral analyses are incorporated, using discrete modelling of the chimney. The reliability of the chimney against the vortex-induced forces is determined using Vanmarck's double barrier crossing analysis. The influence of critical parameters used in both methods is discussed. These parameters are Strouhal number, lift coefficient, correlation length, turbulence intensity, and total damping.

Keywords: Direct spectral analysis, modal analysis, reliability analysis, vortex-induced vibration

1. INTRODUCTION

The vortex shedding plays a dominant role in developing the across-wind force. Thus analyzing the structure across wind vibration is difficult. Contrarily, many analytical approaches have been developed to analyze the along-wind response of the structures. Unlike along wind response, where the buffeting mechanism is the foremost source of excitation, the across-wind vibration occurs due to the involvement of several excitations. The primary excitation occurs due to pressure differences arising from fluctuating shedding of vortices from either side of the structures. The phenomenon is known as vortex shedding. Another excitation source is the buffeting force from the fluctuating lateral element of turbulence in the incoming flow. As long as the amplitude of vibration of the circular chimney is small, the vortex-induced vibration can be examined within the framework of linear random vibration theory. Vickery and Basu, 1972 developed a simple expression to obtain the response of slender structures of circular cross-section using quasi-static theory, which shows a good agreement between measured and computed response.

On carefully investigating the studies conducted on chimneys, it is safe to conclude that gustiness and uncertainties associated with wind loading and structural characteristics lead to variability in the response of wind-sensitive structures. The chances of predicting the dynamic response of a structure without incorporating the variability arising from uncertainties can lead to the failure of structures. Therefore, to ensure the safety of the structure, a probabilistic assessment of the structure is necessary. Vanmarck, 1975 double barrier crossing analysis is presented in the paper. The chimney is analyzed on frequency domain using direct spectral method and modal method. Various parameters associated with the direct spectral and modal analyses are explained and examined. These parameters are Strouhal number, total damping, aerodynamic damping, lift coefficient and correlation length. The problems include tapered chimneys of height 210m with fundamental frequency, top and bottom diameter as 0.33Hz, 10m and 20m, respectively.

2. THEORY

Direct spectral analysis: The dynamic equation of motion represents the motion of the structure vibrating in n^{th} mode experiencing vortex shedding lift force, $F_{Ln}(t)$. The motion of the equation of a circular chimney can be expressed as

$$M\ddot{Y} + (C_s + C_{ae})\dot{Y} + KY = F_L(t) \tag{1}$$

M and K is the diagonal mass and stiffness matrix of the vibrating structure considering translational mode, C_s is the Rayleigh damping, C_{ae} is the aerodynamic damping. The PSDF of the vortex force (F_{Li}) is expressed as $S_{LL,t}$.

$$\frac{S_{LL,t}.\,\omega}{\sigma_{LL,t}^2} = \frac{\omega}{\sqrt{\pi}.\,\omega_s.\,B_t}.\,exp\left(-\left(\frac{1-\frac{\omega}{\omega_s}}{B_t}\right)^2\right) \tag{2}$$

Where $\sigma_{LL,t}^2$ and B_t are the lift force coefficient and spectral bandwidth due to translational vortex force, respectively. The PSDF response (S_{yy}) of a structural system in its n^{th} vibration mode is governed by the frequency response function (H_L), given in Eqn. (4).

$$S_{yy} = H_L.\,S_{FF}.\,H_L^{*T} \tag{3}$$

$$H_L = [K - M\omega^2 + i(C_s + C_{ae})\omega]^{-1} \tag{4}$$

Mean peak displacement is obtained as $y = k_p\sigma_t$, in which k_p is the Davenport, 1963 peak factor.

Modal analysis: In modal analysis, the normalized motion of the equation of a circular chimney is be expressed as

$$\overline{m}_i\ddot{z}_i + \bar{c}_i\dot{z}_i + \bar{k}_i y_i = \bar{f}_{Li}(t) \tag{5}$$

where, $\overline{m}_i = \phi_i M \phi_i^T$; $\bar{c}_i = \phi_i(C_s + C_{ae})\phi_i^T$; $\bar{k}_i = \phi_i K \phi_i^T$; $\bar{f}_{Li}(t) = \phi_i^T F_{Li}$ All the procedure remains the same as for direct spectral analysis. The response then expressed as

$$\sigma_y^2 = {\phi_i}^T H_L(f).\,S_{Ln}(f_n).\,H_L^{*T}(f)\,\phi_j \tag{6}$$

Reliability analysis: The probability of failure can be formulated as

$$P_f = \int 1 - (L_0 e^{-\alpha_D T}) \tag{7}$$

Where L_0 is the probability of the starting of the threshold given as, $L_0 = 1 - e^{-\frac{\psi^2}{2}}$, α_D is the double barrier decay rate, which is given as $\alpha_D = 2\gamma_{M0}$ and T is the duration of the wind force acting on the structure.

3. RESULT AND DISCUSSION

Direct Spectral and Modal analysis: Response obtained from the modal analysis are similar to the direct spectral analysis. Therefore, for taller chimneys, the modal analysis should be opted for obtaining the responses.

Table 1. Critical nondimensional response of chimney considering the direct spectral and model analysis including aerodynamic damping

	Direct Spectral Analysis	Modal Analysis
RMS value	0.0077	0.0078
Peak value	0.0307	0.0313

3.1. The influence of parameters

Turbulence intensity: Turbulence in incoming flow has a crucial effect on stationary and vibrating chimneys because it decreases the lift forces acting along the chimney's axis and broadens the lift spectrum's bandwidth. High turbulence intensity destroys the vortex shedding cell, leading to lower vibrational amplitude, as seen in Fig. 1. Higher turbulence intensity showed a lesser amplitude of vibration and vice versa.

Correlation length: The fluctuating lift forces acting on one section along the structure is imperfectly correlated with lift forces at different section. This correlation is symbolized by a term known as correlation length, expressed in terms of the number of diameters. Higher the correlation length, higher the amplitude of vibration, as seen in Fig. 2.

Total damping: Structure with low structural damping generates a high vibrational amplitude of vibration while experiencing vortex shedding (Fig. 3). It can be observed from Fig. 3 that for taller chimneys like 210m, the effect of structural damping (ζ_s) is more promising because as the amplitude of vibration increases, aerodynamic damping decreases, which means it is only structural damping that is affecting the response.

Strouhal number: The effect of the Strouhal number appears to be significantly less on the amplitude of vibration of the chimney. Although as the value of the Strouhal number increases, there is an increment in the amplitude of vibration, but the increment is so low that it cannot be depicted in Fig. 4.

Lift coefficient: The more immense the strength of vortex shedding, the larger the lift force coefficient and thus significant response of chimney is observed for all the chimney cases in Fig. (5). Also, for the taller chimney, the amplitude of vibration will be more.

Reliability analysis: The increase in Strouhal number does not represent an increase in peak amplitude of vibration observed in the reinforced concrete chimney. Thus, the effect of Strouhal number variation on the chimney response is entirely intractable (Fig. 9). Similarly, the analysis of the chimney subjected to the range of turbulent intensity indicates that the failure is more likely to occur at a lower range value (Fig. 6). Higher the correlation length, higher the peak displacement of the chimney, causing more probability of failure, as shown in Fig. 7. A similar trend can be observed for the coefficient of lift force, where Fig. 10 accurately defines the behavior of the lift coefficient. The reliability of the structure is sensitive to the total damping as shown in Fig. 8.

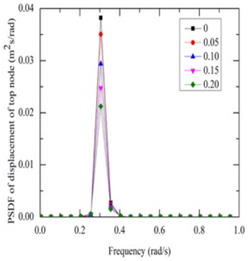

Figure 1. PSDF of the tip of displacement of the chimneys for different turbulence intensities (I_v).

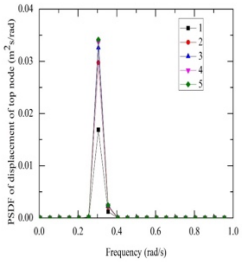

Figure 2. PSDF displacement of tip of chimney considering various correlation lengths.

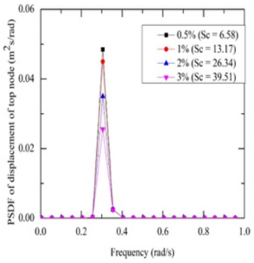

Figure 3. PSDF displacement of tip of chimney considering various total damping.

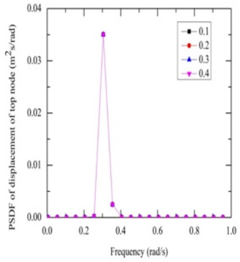

Figure 4. PSDF displacement of tip of chimney considering various Strouhal numbers.

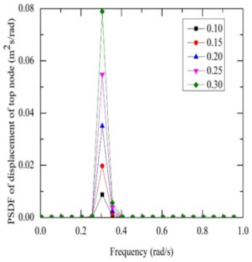

Figure 5. PSDF displacement of tip of chimney for different lift coefficients.

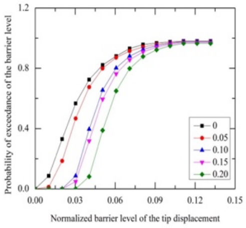

Figure 6. The effect of turbulence intensity on the barrier crossing.

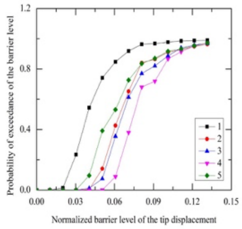

Figure 7. The effect of correlation length on the barrier crossing.

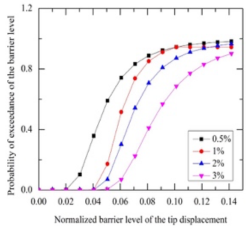

Figure 8. The effect of total damping on the barrier crossing.

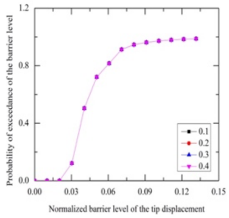

Figure 9. The effect of Strouhal number on the barrier crossing.

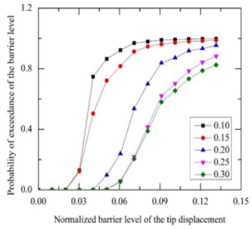

Figure 10. The effect of lift coefficient on the barrier crossing.

4. CONCLUSIONS

After performing direct spectral and modal analysis on the tapered chimney of height 210m, the following conclusions can be observed.

- Turbulence intensities in the incoming flow plays a vital role in the determination of the response of circular chimney. High turbulence intensity destroys the vortex cell causing reduction in lift force which in turn leads to broadens the bandwidth and hence amplitude of vibration decreases.
- With increase in Strouhal number, the response of the structures increases but the increment is not much noticeable.
- Correlation length effective influences the response of the structure. As the correlation length increases, large displacement is observed because lift force is perfectly correlated for a specified correlation length.
- Lift force is expressed in terms of lift coefficient for the frequency domain analysis. Higher the lift force, higher will be the lift coefficient which increase the amplitude of vibration.

5. REFERENCES

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